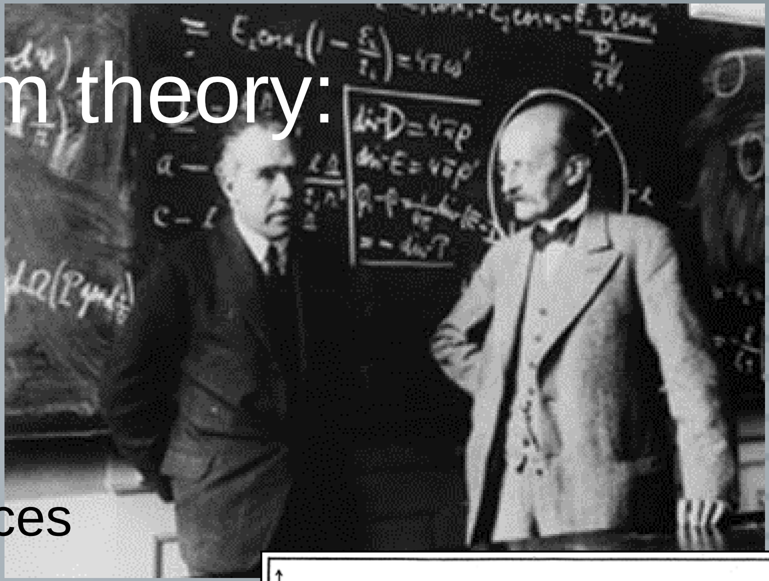
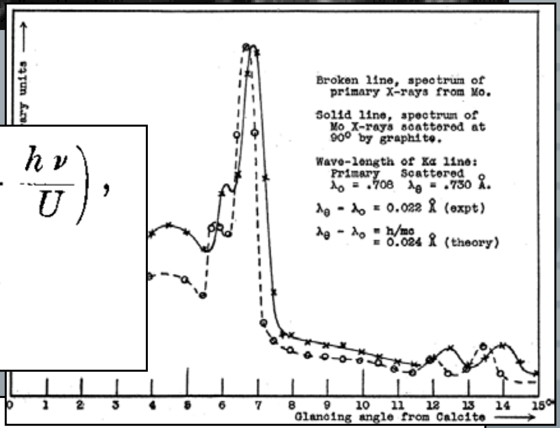
The history of early quantum theory: Myths and facts



Oliver Passon
University of Wuppertal
School for Mathematics and Natural sciences
Physics education group



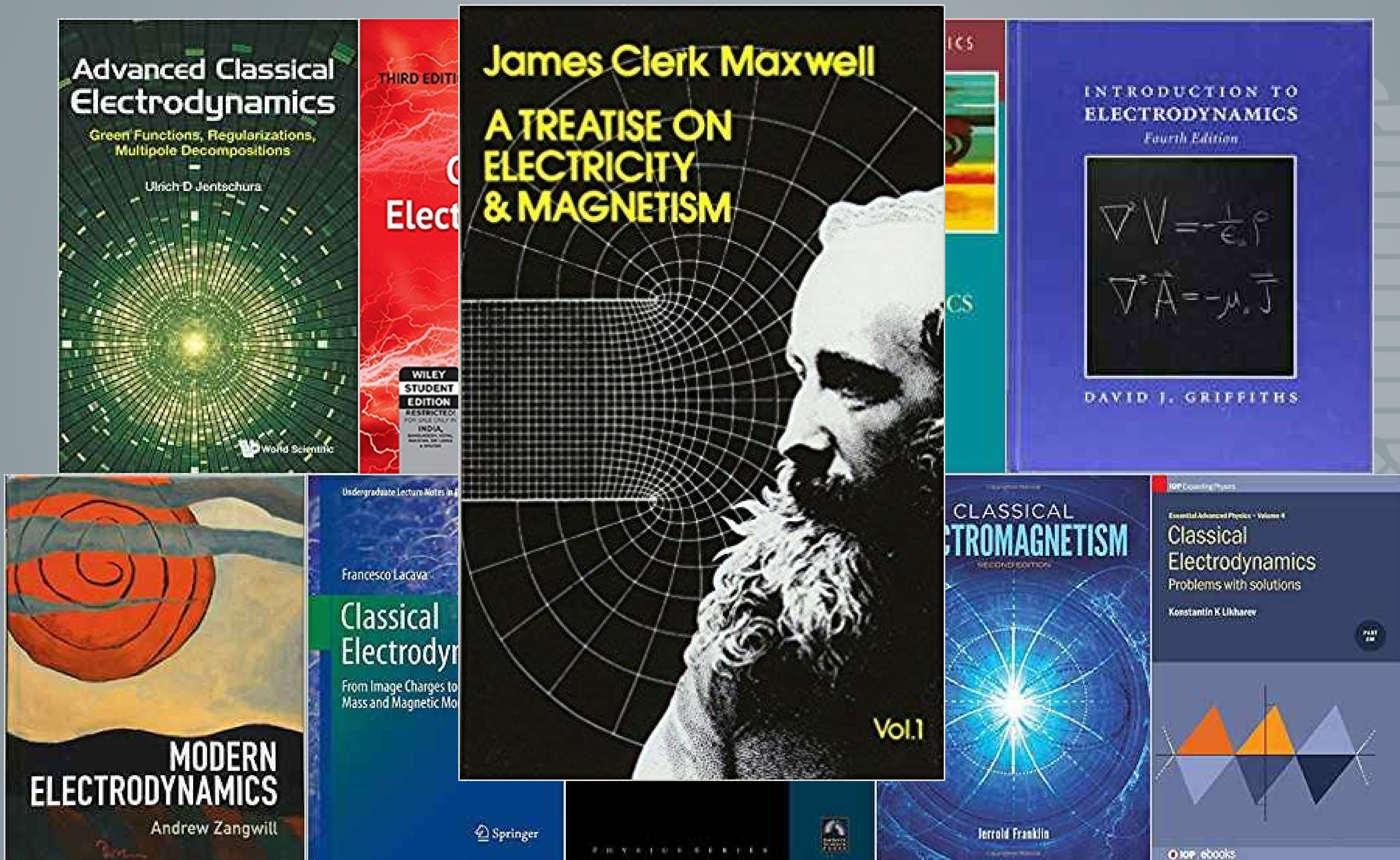
$$\frac{1}{\vartheta} = \frac{k}{h\nu}\log\left(1+\frac{h\nu}{U}\right),$$

$$U = \frac{h\nu}{e^{\frac{h\nu}{k\vartheta}}-1}$$

$$W = \left(\frac{v}{v_0}\right)^{\frac{N}{R}\frac{E}{\beta\nu}}.$$

$$\nu = \frac{\pi^2 m e^2 E^2}{2h^3}\left(\frac{1}{f^2(\tau_2)} - \frac{1}{f^2(\tau_1)}\right).$$

Physics and the History of Physics



Physics and the History of Physics

Physics is traditionally taught ahistorically

- No primary sources
- No dealing with “classical positions”

To put it more positive

Physics teaching (on school and university level) aims at conveying

- Knowledge and
- Scientific practice of current theories

However: The teaching is often framed historically

- Historical notes in the introductions, side notes
- Laws and equations are “historically” labeled („Newton’s law”, “Huygen’s principle”, …)
- Questions of priority play a major role (→reward system)

Clearly, the quality of these historical notes is rather poor…

However, our concern is not with minor simplifications, inaccuracies etc.

Phys. Ed. Vol. 14 1979

History and quasi-history in physics education—part 1

M A B WHITAKER
Department of Physics, New University of Ulster, N. Ireland

- „Whig history“: history as purposeful development to the current state of knowledge (“success story”)
- Selection of topics which are of current interest still
- Suppression of…
  - …the original motives
  - …social and economical conditions
  - …blind alleys of research
- „rational reconstruction” which claims to have actually happened.

Examples:

- Galilei‘s alleged experiments at the leaning tower of Pisa
- The often claimed relation between the *Michelson-Morley ether-drift-experiment* and Einstein‘s discovery of SR

Reasons for the origin of quasi-history:

July, 13.-17. 1970 , MIT: *International Working Seminar on the Role of the History of Physics in Physics Education*.

1. The use and abuse of historical teaching in physics*
by Martin J. Klein

Science Vol 183 1974

Should the History of Science be Rated X?
The way scientists behave (according to historians) might not be a good model for students.

Stephen G. Brush

Science & Education 1, 349–363, 1992.

A Sense of History: History of Science and the Teaching of Introductory Quantum Theory

HELGE KRAGH*

Why quasi-history?

- **M. J. Klein**: ***structural reasons*** for the distortion since “physics” and “history of physics” have different goals → history of physics should not be mingled into physics teaching!

- **M. A. B. Whitaker**: ***didactical reasons*** for the distortion. Quasi-historical accounts bring the scientific facts into an order which can be easily remembered for examination purpose!

- **S. G. Brush**: ***Ideological reasons*** for the distortion. Quasi-history produces a sense of participation in a social and methodological tradition (“neutral factfinder…“) which is often fictious.

Note: More than one reason could be present……

Working definition of quasi-history

„quasi-history“	=	historical narrative according to which physics proceeds in rational discourse and based on evidence

„poor history – but good physics“

„The“ quasi-history of (early) quantum theory

1. Quantum theory originated from Planck‘s explanation of black-body radiation in 1900. In order to prevent the UV-catastrophe at high frequencies he introduced discontinuous energies ($\varepsilon = h\upsilon$).
2. Einstein, in 1905, applied Planck’s idea on light; introduced the light-quantum and explained the photo-electric effect. This effect can not be explained with a wave theory of light.
3. Bohr’s model in 1913 combined both, Planck’s and Einstein’s insides: electrons move on discrete orbits and radiate/absorbe photons upon transition between energy levels.
4. Compton’s discovery and explanation (1922/23) of the effect named after him (“wave length shift of X-rays upon scattering on electrons”) provides the unquestionable proof of the light-quantum-hypothesis.

= „right“ = ambiguous = disputed = „wrong“

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How „quasi-historical“ is this account..?

- **Line of tradition:** energy quantum→ light quantum → explanation and prediction → experimental confirmation

- Anomaly as trigger for new theories („UV catastrophe“, „photo-electric effect“)

- Myth of the unquestionable experimental evidence (Compton effect)

- Complete neglect of social, economical or political conditions…

Ad 1) **black-body radiation, Planck and the quantization of energy**

1. Quantum theory originated from Planck's explanation of black-body radiation in 1900. In order to prevent the UV-catastrophe at high frequencies he introduced discontinuous energies ($\varepsilon = h\upsilon$).

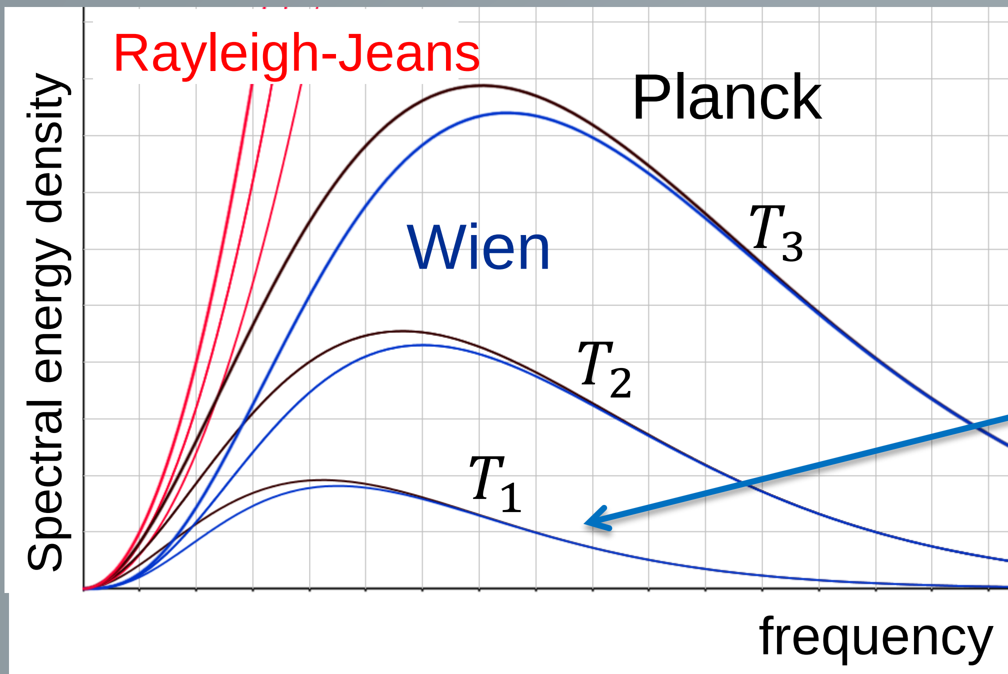
= „right" = ambiguous = disputed = „wrong"

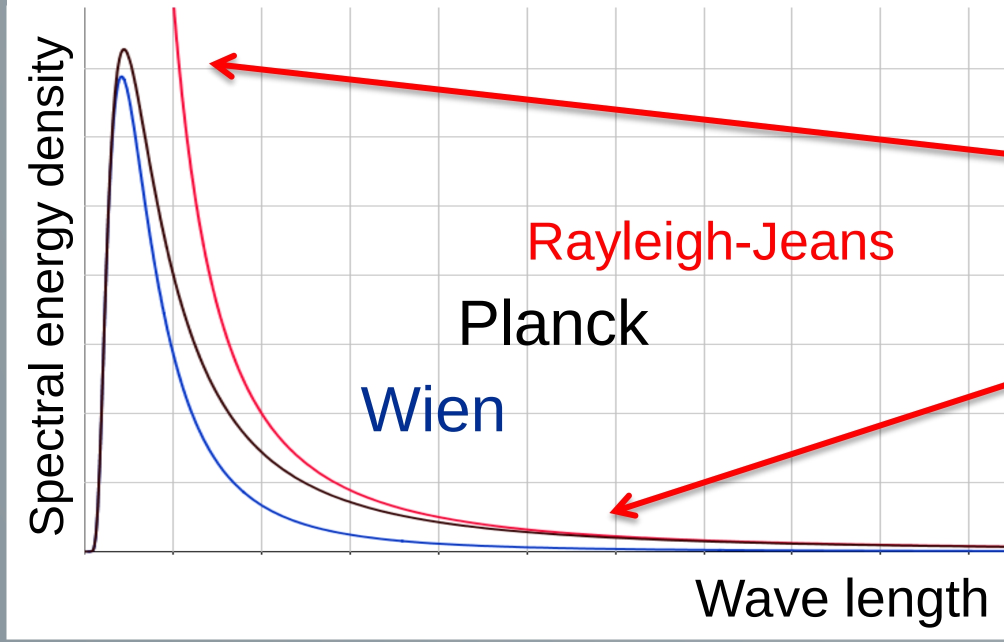
Black body = idealized body which absorbs all

Kirchhoff (1859): spectral energy density u should be given by an universal function $u(T, \mathrm{v})$

Wien(1893) : $u(T, \mathrm{v}) = \mathrm{v}^3 \cdot f\left({}^{\mathrm{v}}/_{T}\right)$ (Wien‘s displacement law)

Remark:
$u(T, \mathrm{v})$ denotes the radiation energy per volume and frequency interval, i.e. has the unit $\frac{Js}{m^3}$





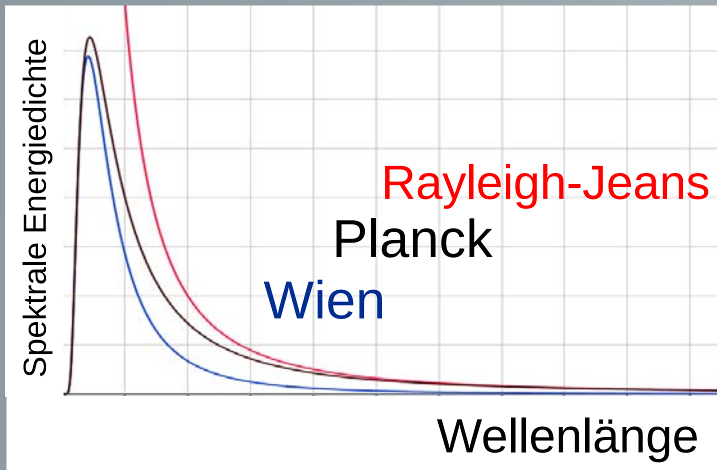
Wien describes the small wave length range ($h\nu \gg kT$)

R-J diverges for small wavelength (UV catastrophe)

R-J describes the large wave length range ($h\nu \ll kT$)

The R-J law follows from the equipartition theorem

Did Planck interpolate the laws of Wien und Rayleigh-Jeans?



$$u_{\mathrm{R-J}}(\nu, T) = \frac{8\pi\nu^2}{c^3} kT \qquad (1905)$$

$e^x \approx 1 + x$ bei $x \ll 1$

$$u_{\mathrm{Planck}}(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \qquad (1900)$$

$e^x - 1 \approx e^x$ bei $x \gg 1$

$$u_{\mathrm{Wien}}(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{\frac{h\nu}{kT}}} \qquad (1896)$$

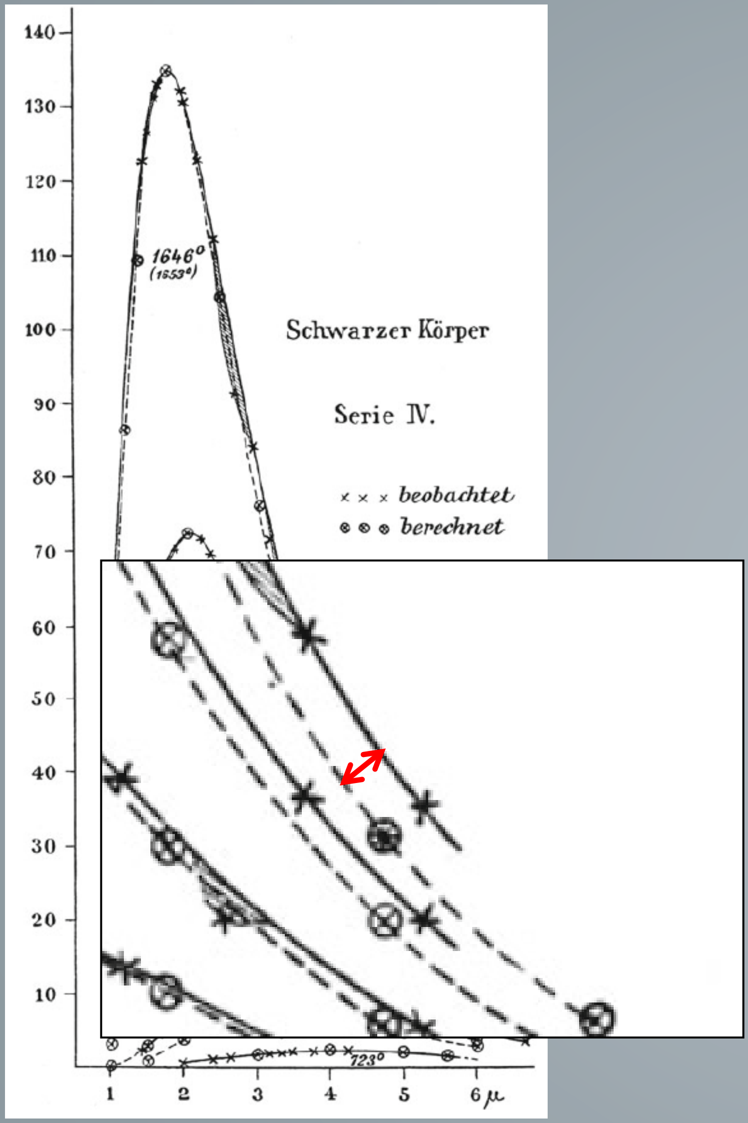
Wien‘s law can only be understood quantum mechanically

Planck was not concerned with the UV-divergence. E. g. the failure of the Dulong-Petit rule compromised the equipartition theorem at that time.

*M. J. Klein (1962) Max Planck and the beginnings of quantum theory. Ar. Hist. Ex.Sci.***1***: 459-479.*

Planck's route to his law of BB radiation

- Modeling the body by charged oscillators
- Proof, that the mean energy of an oscillator with frequency ν is given by: $$u(T,\nu) = \frac{8\pi\nu^2}{c^3}\cdot E(T,\nu)$$ (1899)
- Solve for the energy via the relation: $\frac{dS}{dE} = \frac{1}{T}$
- In 1899 Planck applied an *ad hoc* entropy function which lead to the Wien law (in this context the "quantum of action" h was already introduced as fundamental constant of nature!).



- Measurements of Lummer and Pringsheim indicated a failure of Wien‘s law already in 1899.
- In 1900 Rubens und Kurlbaum extended the measurement range up to 51.2μ (at T=1773K). They found at large wave length: $u \sim T$.
- Planck modified his entropy function and yielded a new radiation law (October 1900 – no physical justification yet) :

$$u(T,\nu) = \frac{8\pi\nu^2}{c^3} \cdot \frac{b\nu}{\exp\left(\frac{a\nu}{T}\right) - 1}$$

Aus: Lummer O., Pringsheim E. (1899) Die Vertheilung der Energie im Spectrum des schwarzen Körpers und des blanken Platins. Verhandlungen der Deutschen Physikalische Gesellschaft 1(12):215–235

On December 14, 1900 Planck presents a physical justification for his new law:

- He knew the result and could work backwards (I. e., motivating the entropy function which yields via $\frac{dS}{dE} = \frac{1}{T}$ the searched for $u(\mathrm{v}, T)$

$$S_\nu = k \ln \frac{\left(1 + \frac{U}{h\nu}\right)^{\left(1 + \frac{U}{h\nu}\right)}}{\left(\frac{U}{h\nu}\right)^{\left(\frac{U}{h\nu}\right)}} \simeq k \ln \frac{\left(1 + \frac{U}{h\nu}\right)!}{\left(\frac{U}{h\nu}\right)!}$$

- By reference to Boltzmann (1877): $S = k \log W$
- One is looking for the number of possibilities to distribute the energy E on N resonators.

If E is considered to be a continuously divisible quantity, this distribution is possible in infinitely many ways. We consider, however—this is the most essential point of the whole calculation—E to be composed of a very definite number of equal parts and use thereto the constant of nature $h = 6{\cdot}55 \times 10^{-27}$ erg sec.

This constant multiplied by the common frequency ν of the resonators gives us the energy element ε in erg, and dividing E by ε we get the number P of energy elements which must be divided over the N resonators.

If the ratio is not an integer, we take for P an integer in the neighbourhood.

From the theory of permutations we get for the number of all possible complexions

$$\frac{N(N+1).(N+2)\dots(N+P-1)}{1.2.3\dots P} = \boxed{\frac{(N+P-1)!}{(N-1)!\,P!} = W}$$

or to a sufficient approximation,

$$= \frac{(N+P)^{N+P}}{N^N P^P}.$$

Proof (Kamerlingh Onnes and Ehrenfest 1914):
View the distribution of P energy elements ε on N oscillators as a character string: $\varepsilon\varepsilon|\ |\varepsilon\varepsilon|\ \varepsilon$ (here: $N = 4, P = 5, E = 5\varepsilon$)

Such a string contains P times the symbol „ε“ and $N - 1$ times the symbol „|“. Hence, $(N + P - 1)!$ permutations can be build. However, the exchange between the symbols lead to a equivalent distribution. In order to avoid this double counting one needs to divide by $(N - 1)!\, P!$

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This entropy function leads to the radiation law

$$u(T,\nu) = \frac{8\pi\nu^2}{c^3} \cdot \frac{\epsilon}{\exp\left(\frac{\epsilon}{kT}\right) - 1}$$

Function of $\frac{\varepsilon}{kT}$; Wien‘s displacement law implies $u(\nu/_T) \to \varepsilon = h\nu$

Is this the hour of birth of quantum theory?

T S Kuhn (1978) „Black-Body Radiation and the Quantum Discontinuity“ UCP: Chicago.

The two readings of the relation

$$W = \frac{(P+N-1)!}{(N-1)!P!}$$

i. **Discontinuous reading:** The equation gives the number of permutations to divide P discrete energy elements on N resontors. This suggests a discrete emission and absorption process.

ii. **Continuous reading:** The equation gives the number of permutations to divide N resonators on discrete energy cells (taking energy conservation into account).Within each cell the resonator may be placed arbitrarily. This suggests that emission and absorption process are continuous still.

This ambiguity makes plausible why Planck‘s law – experimentally well confirmed – did not trigger a debate on quantization…

Ad 2) **Einstein, the light quantum and the photo electric effect**

2. Einstein, in 1905, applied Planck's idea on light; introduced the light-quantum and explained the photo-electric effect. This effect can not be explained with a wave theory of light.

= „right" = ambiguous = disputed = „wrong"

On a Heuristic Point of View about the Creation and Conversion of Light†

A. EINSTEIN

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- Einstein quotes Planck‘s law in § 2 (of 9) – but only to show that for $\lambda \to \infty$ the Rayleigh-Jeans law is recovered…
- His own argument is completely based on Wien’s law.
- He considers radiation confined in a volume V_0 and calculates the probability of a fluctuation into a sub-volume V. The result is compared to the fluctuation probability of a gas consisting of n particles:

$$W = \left(\frac{v}{v_0}\right)^n$$

$$W = \left(\frac{v}{v_0}\right)^{\frac{N}{R}\frac{E}{\beta\nu}}$$

Corresponds to $E/h\nu$

Monochromatic radiation of low density behaves—as long as Wien's radiation formula is valid—in a thermodynamic sense, as if it consisted of mutually independent energy quanta of magnitude $R\beta\nu/N$.

7. On Stokes' Rule

8. On the Production of Cathode Rays by Illumination of Solids

9. On the Ionisation of Gases by Ultraviolet Light

Einstein 1906 on Planck' energy quanta and his own light quanta

ON THE THEORY OF LIGHT PRODUCTION AND LIGHT ABSORPTION

by A. Einstein

[*Annalen der Physik* 20 (1906): 199-206]

In a study published last year[1] I showed that the Maxwell theory of electricity in conjunction with the theory of electrons leads to results that contradict the evidence on black-body radiation. By a route described in that study, I was led to the view that light of frequency ν can only be absorbed or emitted in quanta of energy $(R/N)\beta\nu$, where R denotes the absolute constant of the gas equation applied to one gram-molecule, N the number of actual molecules in one gram-molecule, β the exponential coefficient of Wien's (and Planck's) radiation formula, and ν the frequency of the light in question. This relationship was developed for a range that corresponds to the range of validity of Wien's radiation formula.

At that time it seemed to me that in a certain respect Planck's theory of radiation[2] constituted a counterpart to my work. New considerations, which are being reported in §1 of this paper, showed me, however, that the theoretical foundation on which Mr. Planck's radiation theory is based differs from the one that would emerge from Maxwell's theory and the theory of electrons, precisely because Planck's theory makes implicit use of the aforementioned hypothesis of light quanta.

- Einstein follows the “discontinuous reading”
- Einstein suggests that his light quanta correspond to Planck’s energy quanta

XXXIII. *Simplified Deduction of the Formula from the Theory of Combinations which Planck uses as the Basis of his Radiation Theory. By* P. Ehrenfest *and* H. Kamerlingh Onnes *.

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APPENDIX.

The contrast between Planck's hypothesis of the energy-grades and Einstein's hypothesis of energy-quanta.

As a matter of fact, Planck's energy-elements were in that case almost entirely identified with Einstein's light-quanta, and accordingly it was said that the difference between Planck and Einstein consists herein, that the latter assumes the existence of mutually independent energy-quanta also in empty space, the former only in the interior of matter, in the resonators.

	A	*B*	*C*
I	1	1	1
II	1	1	2
III	1	2	1
IV	1	2	2
V	2	1	1
VI	2	1	2
VII	2	2	1
VIII	2	2	2

Example.: $P = 3$ elements on $N = 2$ resonators

Einstein: $2^3 = 8$ distinguishable distributions

Planck: $\frac{(3+2-1)!}{(3)!(2-1)!} = 4$ distinguishable distributions

We may summarize the above as follows :—Einstein's hypothesis leads necessarily to formula (α) for the entropy and thus necessarily to Wien's radiation-formula, not Planck's. Planck's *formal device* (distribution of P energy-elements ϵ over N resonators) *cannot be interpreted in the sense of Einstein's light-quanta.*

Einstein's distinguishable and localized light quanta were modeled after the kinetic theory of gases. They do not correspond to the "photons" as understood currently.

As radical as Einstein's light quantum hypothesis was – in this respect it was too classical still and its rejection more rational than usually assumed.

Until the discovery of the Compton effect nobody believed in light quanta anyway…

Ad 3) and 4): Bohr’s atom and Compton’s effect

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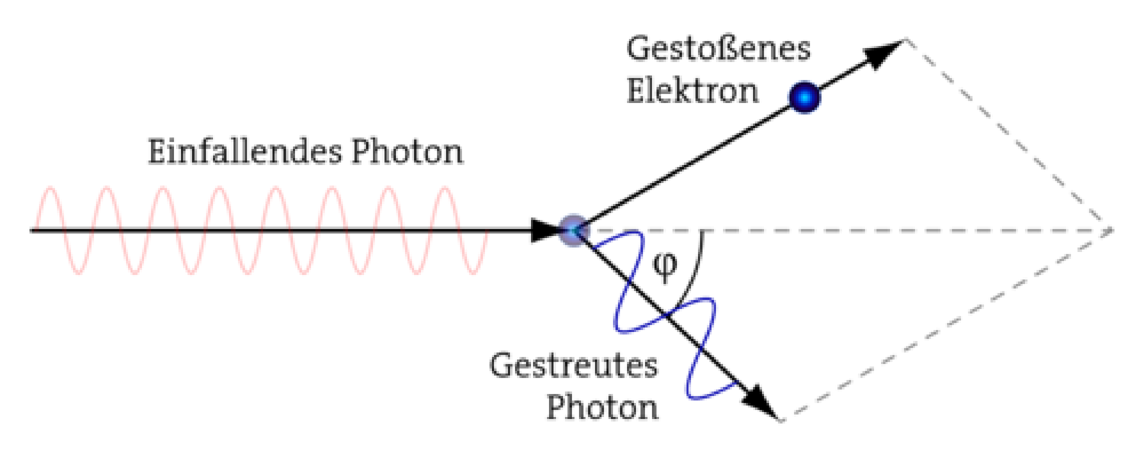
The Compton effect and the particle nature of light

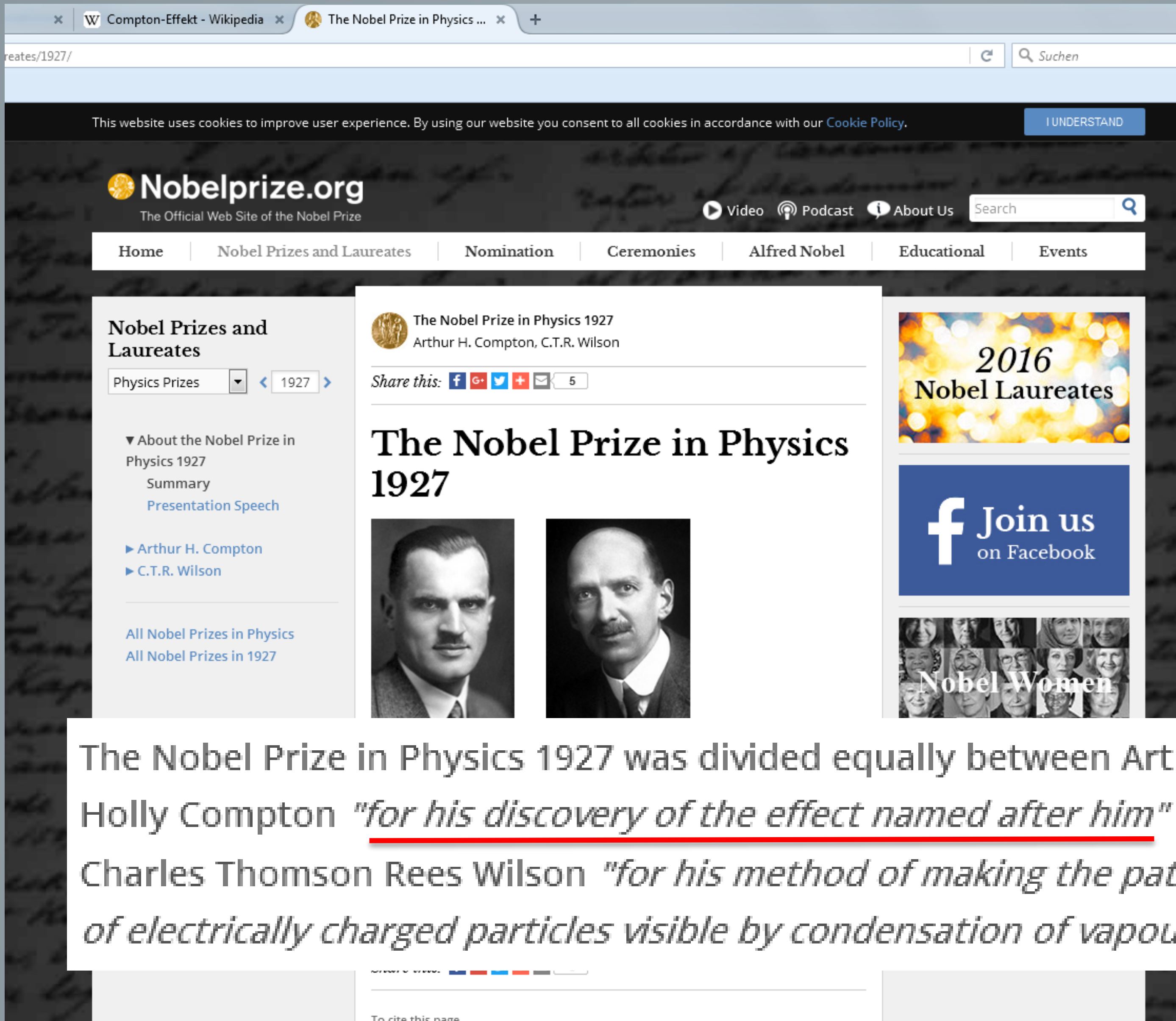


$$\Delta\lambda = \frac{h}{m_{0e}c}(1 - \cos\varphi)\,.$$

Mechanical scattering of point like photons on electrons (1923 – two years before the discovery of quantum mechanics…)

"It was Compton who suggested the name photon for the light quantum. His discovery and explanation of the Compton effect earned him a share of the Nobel Prize in Physics in 1927."

Tipler, P. A. and R. A. Llewellyn (2009) ***Modern Physics***. 5th edition. New York.



The Nobel Prize in Physics 1927 was divided equally between Arthur Holly Compton *"for his discovery of the effect named after him"* and Charles Thomson Rees Wilson *"for his method of making the paths of electrically charged particles visible by condensation of vapour"*.

"[...] the Compton effect has, through the latest evolutions of the atomic theory, got rid of the original explanation based upon a corpuscular theory. The new wave mechanics, in fact, lead as a logical consequence to the mathematical basis of Compton's theory. Thus the effect has gained an acceptable connection with other observations in the sphere of radiation." (Nobel Prize, 1927)

Karl M. G. Siegbahn

9. ***Über den Comptoneffekt;***
von E. Schrödinger

Wir zeigen, daß sich in enger Anlehnung an das oben genannte Brillouinsche Resultat eine wellenmechanische Deutung der Comptonschen Beziehungen geben läßt, welche nicht minder einfach ist als die quantenmäßige Impuls-Energiebetrachtung.

Se also: J. Strnad 1986 „The Compton effect - Schrödinger's treatment“ *Eur. J. Phys*. **7**: 217-221
(Such a **semi-classical** treatment is also possible for the photoelectric effect…)

The kinematic treatment shows how energy and scattering angle relate. For the probability of certain scattering angles one needs the differential cross section:

Über die Streuung von Strahlung durch freie Elektronen nach der neuen relativistischen Quantendynamik von Dirac.

Von **O. Klein** und **Y. Nishina** in Kopenhagen.

(Eingegangen am 30. Oktober 1928.)

$$\frac{d\sigma}{d\Omega}_{\text{Klein-Nishina}} = \frac{1}{2}\frac{\alpha^2}{m^2}\left(\frac{E'}{E}\right)^2\left[\frac{E'}{E}+\frac{E}{E'}-\sin^2\theta\right]$$

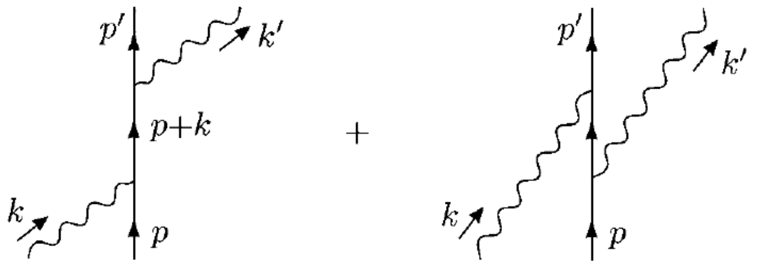
With:

$$\frac{E'}{E} = \frac{1}{1+\frac{E}{m}(1-\cos\theta)}$$

Relativistic and quantum mechanical treatment which includes spin – the radiation field is dealt with semi-classical (i.e. without photons) still!

5.5 Compton Scattering

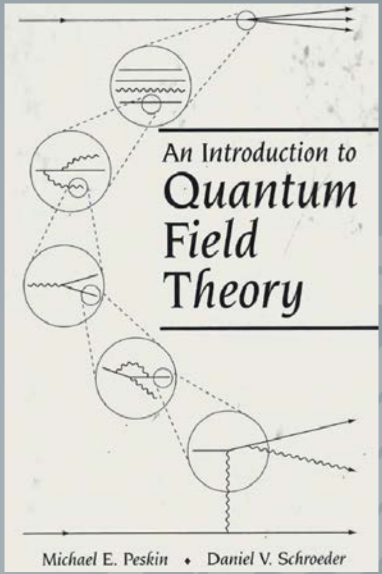
This is our first example of a calculation involving two diagrams:



$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{m^2}\left(\frac{\omega'}{\omega}\right)^2\left[\frac{\omega'}{\omega} + \frac{\omega}{\omega'} - \sin^2\theta\right], \qquad (5.91)$$

where ω'/ω is given by (5.89). This is the (spin-averaged) *Klein-Nishina formula*, first derived in 1929.†

†O. Klein and Y. Nishina, *Z. Physik*, **52**, 853 (1929).



The Klein-Nishina foruma can be derivd in 1. order QED. However, the QED photon is no particle in the “usual sense”…

“It is quite natural to call these excitations *particles*, since they are discrete entities that have the proper relativistic energy momentum relation. (By a *particle* we do not mean something that must be localized in space; $\hat{a}_k^\dagger$ creates particles in momentum eigenstates.) p. 22

Summary I

1. In 1900 Planck introduced the energy element $\varepsilon = h\upsilon$ to account for the black body spectrum. If he intended a physical quantization is debated among historians of physics.
2. Einstein‘s light quantum hypothesis was based on Wien’s law and the analogy with the kinetic theory of gases. His light quanta should not be confused with the current photon concept since they were localized and distinguishable.
3. Bohr’s atomic model applies ideas of Einstein’s theory of specific heat. The light quantum was rejected by Bohr until 1925 or so. In Bohr’s model the radiation follows the frequency condition but is treated classically.
4. The Compton effect convinced many physicists in the reality of light quanta. With the advent of quantum mechanics the picture become more differentiated. Here, Compton- and Photo-effect can be explained with the classical radiation field. A genuine QED effect which could be used to motivate the current photon would be e.g. spontaneous emission.

Summary II

- All arguments for the origin of quasi-historical accounts make an implicit assumption: **The "scientific facts" of the account need to be correct.**

- The quasi-history of early QT violates this condition!
  - The identification of energy quanta and light quanta is wrong
  - Photo- and Compton-effect can be explained without quantization of the radiation field.
  - There is no simple line of tradition which leads from Einstein's light quantum to the current photon concept.

Reasons for these distortions:

- The photo-electric effect can be easily demonstrated in the class room and its "explanation" is so "intuitive"
- The presentation follows partly the reception history…

The "true" quasi history of quantum theory has not been written...